Target space duality and small instanton transitions

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(first paper soon to appear).

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Target space duality

Distler and Kachru hep-th/9707198, Blumenhagen hep-th/9707198 and hep-th/9710021, Blumenhagen and Rahn 1106.4998, Anderson and Feng 1607.04628

- Target space duality is a statement, derived from GLSMs, that two different heterotic compactifications have the same spectrum (and more…)
- How are the two configurations related in terms of target space quantities?:

Calabi-Yau Defining Relations ⟷ Monad Maps

Example:

$$X_D = \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 0 \\ \mathbb{P}^4 & 0 & 5 \end{array} \right]$$

$$0 \to \begin{array}{c} \mathcal{O}(0,-4) \\ \oplus \\ \mathcal{O}(-1,-5) \end{array} \to \begin{array}{c} \mathcal{O}(0,-1)^{\oplus 4} \\ \oplus \\ \mathcal{O}(0,-1) \oplus \mathcal{O}(0,-4) \end{array} \to V_D \to 0$$

is target space dual to:

$$X_R = \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^4 & 1 & 4 \end{array} \right]$$

$$0 \to \begin{array}{c} \mathcal{O}(0,-4) \\ \oplus \\ \mathcal{O}(-1,-5) \end{array} \to \begin{array}{c} \mathcal{O}(0,-1)^{\oplus 4} \\ \oplus \\ \mathcal{O}(-1,0) \oplus \mathcal{O}(0,-5) \end{array} \to V_R \to 0$$

- Question: What is the relation between these two theories from a space time point of view?

- In this talk we will focus on cases like this one where manifolds related by conifolds.

- The transition of the geometry:

Smooth quintic

$$X_D = \left[\ \mathbb{P}^4 \ \middle|\ 5\ \right]$$

Deform complex structure

Nodal quintic

$$X_0 = \left[\ \mathbb{P}^4 \ \middle|\ 5\ \right]$$

Blow up singular points of nodal quintic with (16) $\mathbb{P}^1 s$

$$X_R = \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^4 & 1 & 4 \end{array}\right]$$

The rough process:

- Start on the resolution side for example:

$$V_D$$

$$X_D = \left[\ \mathbb{P}^4 \ \middle|\ 5\ \right]$$

supersymmetric pair creation process of branes (really sheaves) in the gauge and gravitational sectors

$$X_0 = \left[\ \mathbb{P}^4 \ \middle|\ 5\ \right]$$

Sheaves are absorbed via small instanton transitions into the gauge and cotangent bundles to take us to new configuration

$$V_R$$

$$X_R = \left[\begin{array}{c|cc} \mathbb{P}^1 & 1 & 1 \\ \mathbb{P}^4 & 1 & 4 \end{array}\right]$$

Small Instanton Transitions (SITs)

- As we have said, small instanton transitions are going to play a key roll.
- How do we describe small instanton transitions mathematically?:
  - By brute force recombining resolutions of sheaves over curves and bundles (we have this for everything I am going to show you).
  - Or by using (deformations of) Hecke transforms:

$$0 \to V \to V_0 \to F_c \to 0$$

$$0 \to V \to V_0 \oplus \mathcal{O} \to F_c \to 0$$

Aspinwall, Donagi hep-th/9806094 and Ovrut, Pantev, Park hep-th/0001133

The Cotangent Bundles

- We will work on the resolution side of the conifold (so we can see the $\mathbb{P}^1$ s explicitly).
- We will see a transition between bundles here but it isn't really linked to the geometry. That only happens in the singular limit.

Hecke Transform approach:

$$0 \to f^*\Omega_{X_0} \to \Omega_{X_R} \to \mathcal{O}_{\mathbb{P}^1 s}(-2,0) \to 0$$

You can view $f^*\Omega_{X_0}$ as being created when a curve like instanton is absorbed onto Ω_{X_R}.

The Gauge bundles

- So it may seem we now just have to put $\mathcal{O}_{{\mathbb{P}^1}_s}(-2,0)$ into the gauge bundle some how!
- The trouble is that the map you require to make the appropriate transition occur doesn't exist:

$$0 \to V \to V_R \to \mathcal{O}_{{\mathbb{P}^1}_s}(-2,0) \to 0$$

This map is zero.

- Lets look at the gauge bundles in more detail to see more precisely what the proposal is and the details of how the system fixes it.

- The gauge bundles in these target space dual examples always admit rank changing small instanton transitions to emit sheaves associated to rather specific curves.

Deformation side:

$$V_D \longrightarrow \hat{V}_D$$

Deformation

$$0 \to \hat{V}_D \to V_s \oplus \mathcal{O} \to \mathcal{O}_{C_D} \to 0$$

where
$$C_D = \left[\begin{array}{c|ccc} \mathbb{P}^4 & 5 & 1 & 4 \end{array} \right]$$

and
$$0 \to \mathcal{O}(0,-4) \to \mathcal{O}(0,-1)^4 \to V_s \to 0$$

Resolution side:

$$V_R \longrightarrow \hat{V}_R$$

Deformation

$$0 \to \hat{V}_R \to V_s \oplus \mathcal{O} \to \mathcal{O}_{C_R} \to 0$$

where

$$C_R = \left[\begin{array}{c|cccc} \mathbb{P}^1 & 1 & 1 & 1 & 0 \\ \mathbb{P}^4 & 1 & 4 & 0 & 5 \end{array} \right]$$

- The two curves C_R and C_D are special in that they both become divisors in certain limits where we approach the nodal quintic! (we will come back to this later).
- They are also built out of the bits of the gauge bundle that played an active role in target space duality
- The "spectator bundles" will do nothing throughout this process (Candelas, de la Ossa, He and Szendroi 0706.3134).

So – the process at the level of classes:

$$\mathrm{c}_2(\Omega_{X_R}) = \mathrm{c}_2(V_R)$$

Pair create curve supported sheaves ↓ Pair create curve supported sheaves

$$\mathrm{c}_2(\Omega_{X_R}) + [\mathbb{P}^1 s] = \mathrm{c}_2(V_R) + [\mathbb{P}^1 s]$$

SIT in cotangent bundle ↓ SIT in gauge bundle

$$\mathrm{c}_2(f^*\Omega_{X_0}) = \mathrm{c}_2(V_s) + [c_R] + [\mathbb{P}^1 s]$$

↓ "Brane" recombination

$$\mathrm{c}_2(f^*\Omega_{X_0}) = \mathrm{c}_2(V_s) + [c_D]$$

↓ SIT in gauge bundle

$$\mathrm{c}_2(f^*\Omega_{X_0}) = \mathrm{c}_2(V_D)$$

- At level of classes this works great…
- … but the problem is you can't recombine the five brane-like sheaves…
- … and if you can't recombine them they can not be properly absorbed back in to the spectator bundle via a small instanton transition.

- Fortunately the system knows how to solve this. The transition really takes place at the nodal point in moduli space, and there recall that our curves deform to become divisors.

"Brane (really sheaf) recombination"

- We want to write a sequence

$$0 \to \mathcal{O}_{\mathbb{P}^1 s}(-2,0) \to \mathcal{O}_{c_D}(-1,0) \to \mathcal{O}_{c_R} \to 0$$

you need those strange twists for the Chern classes to work out. This looks wrong for the central sheaf, we will discuss this more shortly.

- But computation shows

$$\mathrm{Ext}^1(\mathcal{O}_{c_R}, \mathcal{O}_{\mathbb{P}^1 s}(-2,0)) = 0$$

and the $\mathbb{P}^1$ Supported sheaf cant be absorbed into the bundle unless it is recombined with something.

- However, in the limit associated to the singular manifold where the curves become divisors we have.

$$\mathrm{Ext}^1(\mathcal{O}_{c_R}, \mathcal{O}_{\mathbb{P}^1 s}(-2,0))$$

$$= \mathrm{Ext}^1(\mathcal{O}, \mathcal{O}(-2,0)) = \mathbb{C}$$

Now a divisor.

- However we now have $\mathcal{O}_{C_D}(-1,0)$ on X_0 which is a twist of what we wanted: $\mathcal{O}_{C_D}$.
- But to complete the transition you must deform back to the smooth quintic. Then:

$$\mathcal{O}_{C_D}(-1,0) \to \mathcal{O}_{C_D}$$

Moduli

- Given that this is the process, why do the moduli (for example) of the theory match?
- The moduli of the Hecke transform for one of the gauge bundles:

$$\mathrm{Ext}^1(V,V) = H^1(V_s \otimes V_s^\vee) \oplus \mathrm{Ext}^1(V_s, \mathcal{I}_C) \oplus \mathrm{Ext}^1(\mathcal{I}_C, V_s) \oplus H^0(C, \mathcal{N}_C)$$

(the change in the moduli under deformation to the smooth bundle is understood)

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Stay the same on the two sides of the conifold

(the change in the moduli under deformation to the smooth bundle is understood)

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Changes in exactly the opposite way to the Hodge numbers of the manifolds

(the change in the moduli under deformation to the smooth bundle is understood)

Further Topics

- As a by-product, we now have a new duality between M5-brane theories (see talk by Callum Brodie).
- We are looking at non-conifold cases (for example flops).
- The pair creation effect we discussed could be studied in isolation from target space duality in a simpler setting.
- What can be said using this work about the fate of gauge bundles through geometric transitions more generally in heterotic string theory?
- How does this alter our view of the moduli space of heterotic compactifications?